**Integer Linear Programming: Additional Practice Problems**

*1)* *Managing Consultants.* You manage a small consulting firm with three clients. Minimize the total completion time of the projects by assigning **one** project leader to each client (each leader can only manage **one** client). The project completion times for each client and each project leader are summarized below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Client** | | |
| **Project Leader** | **1** | **2** | **3** |
| **T**erry | 10 | 15 | 9 |
| **C**arl | 9 | 18 | 5 |
| **A**nn | 6 | 14 | 3 |

**GAP words:**

O: Minimize total completion time

D: Which project leader to assign to which client?

C: Each project leader can only manage one client; Each client needs to be assigned to a project leader

**GAP math:**

D: T1 = 1 if Terry is assigned to client 1 and T1=0 otherwise. Similarly for T2, T3, C1, C2, C3, A1, A2, A3

O: Minimize 10T1+15T2+9T3+9C1+18C2+5C3+6A1+14A2+3A3

C:

T1+T2+T3 = 1

A1+A2+A3 = 1

C1+C2+C3 = 1

T1+A1+C1 = 1

T2+A2+C2 = 1

T3+A3+C3 = 1

T1, T2, T3, C1, C2, C3, A1, A2, A3 all binary

*2) Selecting Projects.* The Texas Electronics Company (TEC) is contemplating a research and development program encompassing eight major projects. The company is constrained from embarking on all projects by the number of available scientists (40) and the budget available for projects ($300,000).

Following are the resource requirements and the estimated profit for each project:

|  |  |  |  |
| --- | --- | --- | --- |
| **Project** | **Expense ($000)** | **Scientists Required** | **Profit ($000)** |
| 1 | 60 | 7 | 36 |
| 2 | 110 | 9 | 82 |
| 3 | 53 | 8 | 29 |
| 4 | 47 | 4 | 16 |
| 5 | 92 | 7 | 56 |
| 6 | 85 | 6 | 61 |
| 7 | 73 | 8 | 48 |
| 8 | 65 | 5 | 41 |

1. What is the maximum profit, and which projects should be selected?

**GAP Words**

* Objective: Maximize Profit
* Decision Variables: Which projects to launch
* Constraints: Can only launch 1 of each given project, Max Scientists, Max Budget

**GAP Math**

* Decision Variables: P1, P2, …, P8 as binary (0,1)
* Objective Function: P1 \* 36 + P2 \* 82 + P3 \* 29 + P4 \* 16 + P5 \* 56 + P6 \* 61 + P7 \* 48 + P8 \* 41
* Constraints:
  + Max Scientists - P1 \* 7 + P2 \* 9 + P3 \* 8 + P4 \* 4 + P5 \* 7 + P6 \* 6 + P7 \* 8 + P8 \* 5 <= 40
  + Max Budget - P1 \* 60 + P2 \* 110 + P3 \* 53 + P4 \* 47 + P5 \* 92 + P6 \* 85 + P7 \* 73 + P8 \* 65 <= 300

**Answer:** Max Profit = $199,000 and we select projects 2, 5, and 6

1. Suppose that management decides that projects 2 and 5 are mutually exclusive. That is, TEC should not undertake both. As a result, what is the revised project portfolio and the revised maximum profit?

**New Constraint:** Projects 2 and 5 are mutually exclusive 🡺 P2 + P5 <= 1

**Answer:** Max Profit = $195,000 and we select projects 1, 2, 3, and 7

1. Suppose that management also decides to undertake at least two of the projects involving consumer products. (These happen to be projects 5-8.) As a result, what is the revised project portfolio and the revised maximum profit?

**New Constraint:** Minimum of 2 projects from projects 5 through 8 🡺

P5 + P6 + P7 + P8 >= 2

**Answer:** Max Profit = $191,000 and we select projects 2, 6, and 7

*3) Car rental.* You are a small car rental business, and you must decide how many car categories to offer to your customers. Local laws require you to list all five categories on your website, even if you do not offer all of them: economy, compact, mid-size, standard and full-size. If you decide to offer a category, you incur a fixed cost of $20,000 per category (each category requires separate purchasing and insurance contract negotiations with the manufacturers). The demand for each category is as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Category** | **1 (economy)** | **2 (compact)** | **3 (midsize)** | **4 (standard)** | **5 (full-size)** |
| Demand | 1000 | 2000 | 400 | 800 | 1000 |

If you decide to not offer a category, you must “upgrade”the customer by offering him a car from a higher category. Whenever you” upgrade” a customer, you incur an additional, variable cost: upgrade one level up costs you $50 (that is, it costs you $50 to upgrade someone from economy to compact, $100 from economy to midsize etc). If your objective is to minimize total costs, what categories should be offered?

**GAP Words:**

O: Minimize Cost

D: Whether to rent cars from each category, How much demand from each category to fill by renting cars from each category

C: Minimum demand for each category, linking fixed cost for renting cars in each category

**GAP Math:**

D: R1, R2,…, R5 as binary where R1 = 1 means we rent cars from category 1. F11, F12, F13, F14, F15, F22, F23,…, F55 as continuous variables, where F12 is how many cars demanded from category 1 that we fill as category 2 rentals, etc.

O: 20,000 \* R1 + 20,000 \* R2 … + 0 \* F11 + 50 \* F12 + 100 \* F13 … + 0 \* F55

C:

**Demand:**

F11 + F12 + F13 + F14 + F15 = 1000

F22 + F23 + F24 + F25 = 2000

F33 + F34 + F35 = 400

F44 + F45 = 800

F55 = 1000

**Linking:**

F11 <= M \* R1

F12 + F22 <= M \* R2

F13 + F23 + F33 <= M \* R3

F14 + F24 + F34 + F44 <= M \* R4

F15 + F25 + F35 + F45 + F55 <= M \* R5

**Binary:** R1, R2, R3, R4, R5 are binary

*4)* California Products Company has the capability of producing and selling three products. Each product has an annual demand potential (at current pricing and promotion levels), a variable contribution, and an annual fixed cost. The fixed cost can be avoided if the product is not produced at all. This information is summarized as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Product | Demand | Contribution ($) | Fixed Cost ($) |
| I | 290,000 | 1.20 | 60,000 |
| J | 200,000 | 1.80 | 200,000 |
| K | 50,000 | 2.30 | 55,000 |

Each product requires work on three machines. The standard productivities and capacities are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Hours per 1,000 Unit | | |  |
| Machine | Product I | Product J | Product K | Hours Available |
| A | 3 | 4 | 8 | 1,900 |
| B | 3 | 4 | 6 | 1,900 |
| C | 2 | 3 | 10 | 1,900 |

a). Determine which products should be produced, and how much of each should be produced, in order to maximize profit contribution from these operations.

**GAP WORDS**

* Objective: Maximize profit
* Decision Variables: which product should be produced. How many should be produced
* Constraints: Production should be less than demand; hours limit for machine A, B, C; linking constraints; binary linking variables

**GAP MATH**

* Decision Variables:
  + Production number: I, J, K
  + Production selection (binary): X,Y,Z
* Objective Function:
  + 1.2 \* I + 1.8 \* J + 2.3 \* K – 60k \* X – 200k \* Y – 55k \* Z
* Constraints:
  + I <= 290,000
  + J <= 200,000
  + K <= 50,000
  + (3/1000) \* I + (4 / 1000) \* J + (8/1000) \* K <= 1900
  + (3/1000) \* I + (4 / 1000) \* J + (6/1000) \* K <= 1900
  + (2/1000) \* I + (3 / 1000) \* J + (10/1000) \* K <= 1900
  + I <= M \* X
  + J <= M \* Y
  + K<= M \* Z

**Answer**: Produce 290,000 units of I, 200,000 units of J and 28,750 unit of K. Maximum profit is 459,125

b). Suppose the demand potential for product K were doubled. What would be the maximum profit contribution?

* **Modify Constraint:** Change the demand limit for K: K <= 100,000

**Answer**: produce 290,000 units of I, 0 unit of J and 100,000 units of K. Maximum profit is 463,000